Small games and cognitive discord

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Preliminaries

An agent must apprehend her world before she can make decisions.

Savage (1954) speaks of reducing the *grand world* to a *small world*.

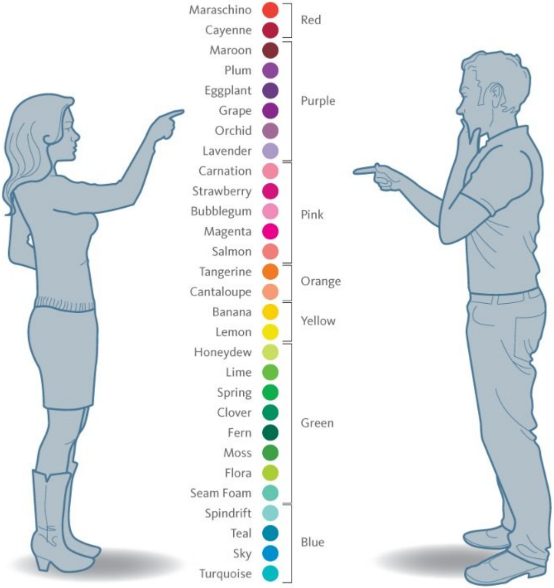
The agent's cognition generates a representation of the environment.

Cognition is not completely under the agent's control.

The agent's cognitive frame distills the grand world into a small world.

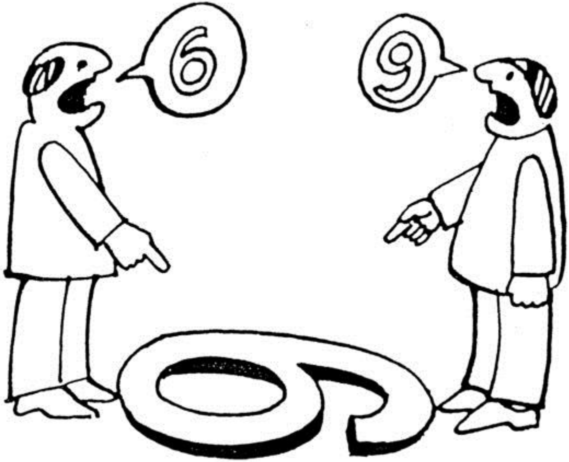
People perceive the world differently.	[**Perception**]
Perception relies on a cognitive frame.	[**Frame**]
Cognition affects behavior.	[**Behavior**]

People perceive differently



Agents rely on cognitive frames

Cognition affects behavior



Main theme

Agents must apprehend the world before making strategic decisions.

The agents' cognitive frames distill the grand world into a small world.

When the game is common knowledge, the players have (miraculously? by assumption?) reduced the grand world to the same small world.

More modestly, agents with a shared frame see the same game and may agree on what is the *obvious way to play* the game (rule of behavior).

Frames may be harmful or beneficial.

Agents with different frames perceive different small worlds: they are in the same strategic situation, but play *different games*.

Open questions:

1) which frames perform better?
2) can agents realize if they are using different frames?
3) how can agents repair a cognitive discord?


I DON'T THINK YOU COMPLETELY GRASP THE SITUATION HERE, FRANCOIS...
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Related literature

–) Heller and Winter (2016, and more) imagine a two-stage game where agents first choose their frame and then play.

Frames equilibrate by some unspecified evolutionary process.

–) Pavan and Tirole (2022) take a similar approach modeling agents' cognitive postures.

Cognition may also be manipulative (espionage, deception).

–) Gibbons, LiCalzi, Warglien (2021) illustrate how a shared frame affects the rule of behavior both in one-shot and repeated interactions.

Concerning different frames, they distinguish incremental discord, when agents apply the same rule of behavior to their perceived game(s), and radical discord, where even their rules of behavior are different.

–) Jehiel (2021) surveys analogy-based expectation equilibrium, where *cognitive types* have access to different aggregate statistics about other players' behavior.

A simple example

> *While the objective game represents the true environment, the subjective model represents the players' perception of their environment.*
>
> Esponda and Souzo, *Econometrica* (2016)

A subjective model is to the objective game what a small world is to the large world.

Consider an objective game with simultaneous moves.

	L	R
$\square$	$3, 1$	$3, 0$
$\bigcirc$	$2, 0$	$0, 5$
$\bullet$	$0, 1$	$2, 2$

Because $\square$ is a dominant action, we predict $(\square, L)$ with payoff $(3, 1)$.

(We assume that the rule of behavior is iterated dominance.)

Suppose both players pay attention to shape, and neglect shade.

They encode $S_1 = \{\square, \bigcirc, \bullet\}$ as the partition $\widehat{S}_1 = \{\square, \widehat{\bigcirc}\}$.

	L	R
$\square$	$3, 1$	$3, 0$
$\bigcirc$	$2, 0$	$0, 5$
$\bullet$	$0, 1$	$2, 2$

$\Longrightarrow$

	L	R
$\square$	$3, 1$	$3, 0$
$\widehat{\bigcirc}$		

	L	R
$\square$	$3, 1$	$3, 0$
$\bigcirc$	$2, 0$	$0, 5$
$\bullet$	$0, 1$	$2, 2$

$\Longrightarrow$

	L	R
$\square$	$3, 1$	$3, 0$
$\widehat{\bigcirc}$	$1, 0.5$	$1, 3.5$

For simplicity, let each choice in $\widehat{\bigcirc} = \{\bigcirc, \bullet\}$ be equally likely.

As $\square$ is still dominant, we predict $(\square, L)$ with payoff $(3, 1)$.

Moving from the objective game to the subjective model does not change the predicted outcome.

Suppose both players pay attention to shade, and neglect shape.

They encode $S_1 = \{\square, \bigcirc, \bullet\}$ as the partition $\widehat{S}_1 = \{\widehat{\bigcirc}, \bullet\}$.

	L	R
$\square$	$3, 1$	$3, 0$
$\bigcirc$	$2, 0$	$0, 5$
$\bullet$	$0, 1$	$2, 2$

$\Longrightarrow$

	L	R
$\widehat{\bigcirc}$	$2.5, 0.5$	$1.5, 2.5$
$\bullet$	$0, 1$	$2, 2$

Now R is dominant, and we predict $(\bullet, R)$ with payoff $(2, 2)$.

The subjective models lead to distinct self-confirming predictions: $(\square, L)$ with shape, and $(\bullet, R)$ with shade.

Because the neglected actions are not played, the subjective model is not challenged.

Neglecting strategies

> *[In a decision problem,] a smaller world is derived from a larger by neglecting some distinctions between states, not by ignoring some states outright.*
>
> L.J. Savage, *Foundations of Statistics* (1954)

Our example derives a smaller game by neglecting some distinctions between actions in the objective game, not by ignoring their existence.

Fog of war refers to uncertainty about one's own capability, as well as adversary capability and intent during military operations.

A major concern is the ability to communicate effectively with people in the field, and ensure that orders are carried out properly.

The space of strategies for a player may be richer than what he perceives or understands or controls.

A *subjective model* coarsens the actions available in the *objective game*.

Coarsening actions

The coarsening of the actions may have different sources:

1) language: we may be unable to describe every opportunity;
(Red subsumes Pantone 185-192-199-200-201-202-215-220 etc.)

2) categorisation: distinct (similar) actions are bundled together;
(enter the market; go West)

3) projection: multidimensional actions are simplified;
(focus on Position: Red-Left is Left, and Blue-Center is Center)

4) cognition: attention is grabbed by a limited number of options;
(one will carefully consider only a few plans)

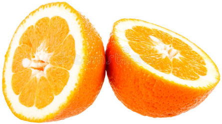
5) decision and execution are allocated to different agents.
(HQ and soldiers on the field)

Barganing over an orange

Two sisters are bargaining over an orange.

Each needs the orange for a cooking recipe.

Let x in $[0, 1]$ be the size of the orange.



Assume linear preferences $u_1(x) = u_2(x) = x$ and $(0, 0)$ as default option.

Nash bargaining predicts a 50-50 split, yielding utility .5 to each sister.

As it turns out, the recipe of the first sister needs orange juice and the recipe of the second sister needs orange peel.

The orange is a vector $\mathbf{x} = (x_1, x_2)$ where x_1 and x_2 in $[0, 1]$ are the (normalized) quantities of pulp and peel.

Preferences are $u_1(\mathbf{x}) = x_1$ and $u_2(\mathbf{x}) = x_2$.

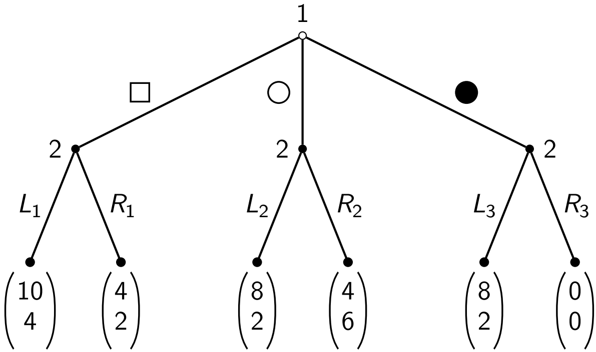
Nash bargaining predicts that 1 gets $x_1 = 1$ and 2 get $x_2 = 1$, yielding utility 1 to each sister.

[This example is commonly attributed to Mary Parker Follett (1868–1933).]

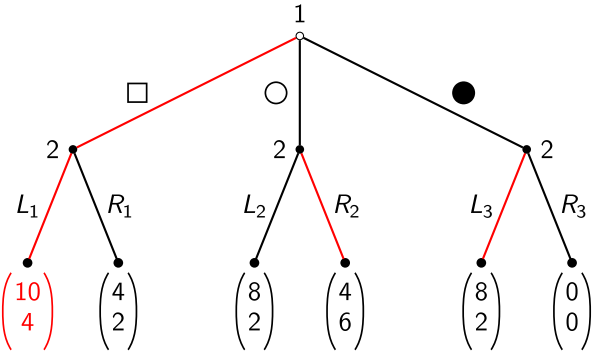
Sequential play

Consider a strategic situation where two agents have identical preferences over all outcomes but one.

Primus moves first and picks one of three choices: a white square (□), a white circle (○), or a black circle (●). Secunda observes his choice and can choose left (L) or right (R).

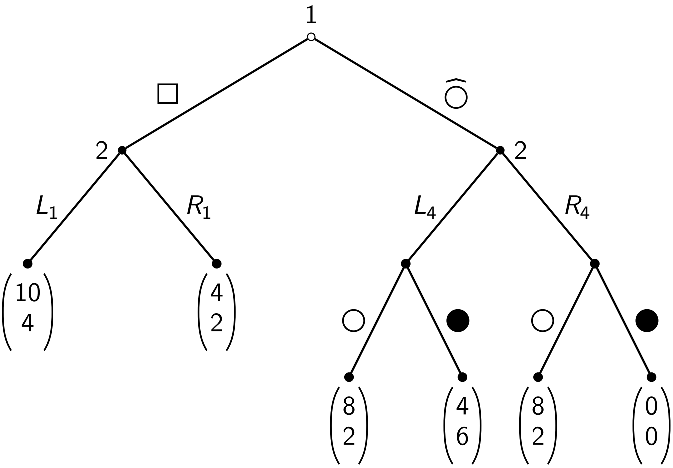


The rule of behavior is backwards induction.



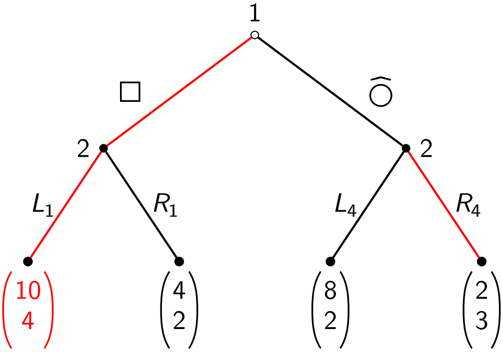
Over the objective game this predicts $(\square, L_1)$ with payoffs $(10, 4)$.

Suppose that both players categorise actions by shape and encode $S_1 = \{\square, \bigcirc, \CIRCLE\}$ as the partition $\widehat{S}_1 = \{\square, \widehat{\bigcirc}\}$.



The final choice is attributed to a random event: $\left(\widehat{\bigcirc}, L_4\right)$ leads to a lottery over $(8, 2)$ and $(4, 6)$ with equal probabilities.

Playing over shape



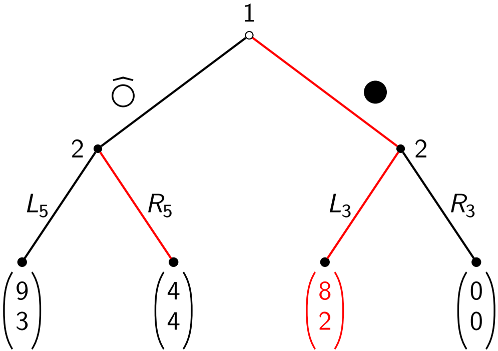
Backward induction predicts $(\square, L_1)$, with payoffs $(10, 4)$.

Players achieve $(10, 4)$ in their (shared) subjective model.

The subjective model is not challenged on the equilibrium path.

Playing over shade

When players categorise actions by shade, they set $\widehat{\bigcirc} = \{\square, \bigcirc\}$.



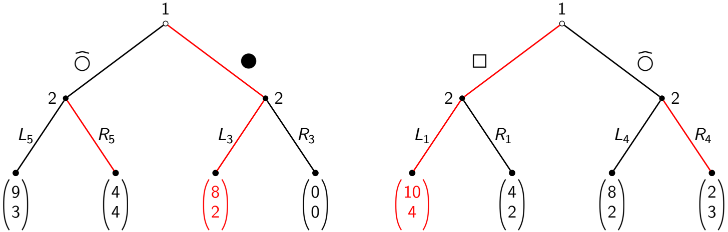
Backward induction predicts $(\bullet, L_3)$, with payoffs $(8, 2)$.

Missed opportunity!

Again, the subjective model is not challenged on the equilibrium path.

Players with different frames

If Primus categorizes by shade and Secunda by shape, they perceive different games.



1 plays ● and 2 (who is puzzled) replies with R.

The profile (●, R) leads to $(0, 0)$, which is Pareto-worst.

Compliments

I'd rather take coffee than compliments just now.
L.M. Alcott, *Little Women*

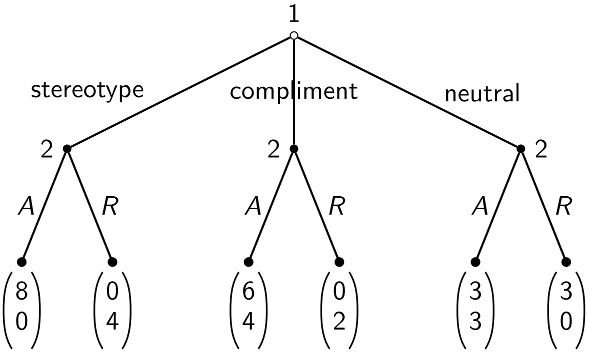
Compliments are nice, but they may not be well-received;
e.g., praising appearance in work-related context.

They may induce negative or sometimes aggressive responses when the compliment is associated to social stereoptypes (even if positive).

In 2006, New York Times columnist Nicholas Kristof published an article (*The Model Students*) extolling Asian Americans for their hard work, ambition, and academic successes: *"increasingly in America, stellar academic achievement has an Asian face."*

Asian Americans were quick to reject this positive depiction, accusing Kristof of making "sweeping generalizations" and "rampant assumptions".

Kristof was surprised by the hostility. A week later he was "still getting indignant e-mails from Asian Americans".



The compliment wil be told and accepted in the objective game.

If will be told and accepted if both players conflate compliment and neutral.

It will never be told if both players conflate stereotype and compliment.

Under misaligned representations it might be told and rejected.

Becoming enemies (McNamara, US SoD)

During the 50s, US gave military support to France during its conflict with Vietnam.

For the US, this was a compensation for France's support to NATO: US would have helped France regardless of US hostility to Vietnam.

Ignoring the geopolitical nature of US policy, Vietnam interpreted US support to France as hostile (neocolonialist).

Vietnam wanted to end the French colonial occupation, and Russia/China were the only countries willing to give them weapons: Vietnam would have accepted their help regardless of a communist political intent.

Ignoring the nationalistic nature of conflict, US interpreted Vietnam behavior as adhesion to the communist block.

Different subjective models led to misreading intentions: eventually the two countries escalated to war.

Formalities

An objective game $G = (N, S, C, f, u)$ defines the sequence

$$N \to S \xrightarrow{f} C \to u$$

where $f : S \to C$ is a function from strategy profiles to consequences.

The mixed extension *expands* each S_i to Σ_i:

$$N \to \Sigma \xrightarrow{f} \Delta(C) \to Eu$$

The subjective model *contracts* at least one S_i into a partition $\widehat{S}_i$ with $|\widehat{S}_i| < |S_i|$, making each $\widehat{S}_i$ the support of a distinct distribution $\widehat{p}$.

For instance, $S_1 = \{\square, \bigcirc, \bullet\}$ contracts to $\widehat{S}_1 = \{\square | \bigcirc, \bullet\}$, with $\widehat{p}(\bigcirc) = \widehat{p}(\bullet) = 1/2$.

The neglected strategies in $\widehat{S}_i$ are replaced by a mixed strategy over $\widehat{S}_i$:

$$N \to \widehat{S} \xrightarrow{f} \Delta(C) \to Eu$$

The complication is that players may be using different partitions.

Projections

A simple way to contract the objective game is to use projections.

For instance, the orange example projects the 2-dimensional pair *(juice, peel)* (x_1, x_2) to the 1-dimensional *share* $\left(\frac{x_1 + x_2}{2}\right)$.

Consider a symmetric 4×4 objective game.

	NW	*NE*	*SW*	*SE*
NW	$5, 5$	$4.80, 5$	$0, 6$	$-.20, 6$
NE	$5, 4.80$	$5.20, 5.20$	$0, 5.80$	$.20, 6.20$
SW	$6, 0$	$5.80, 0$	$1, 1$	$.80, 1$
SE	$6, -.20$	$6.20, .20$	$1, .80$	$1.20, 1.20$

Weak dominance predicts (SE, SE), with payoffs $(1.20, 1.20)$.

Fog of conflict

When both players use vertical projections, they expect:

	N		S	
N	$5, 5$	$4.80, 5$	$0, 6$	$-.20, 6$
	$5, 4.80$	$5.20, 5.20$	$0, 5.80$	$.20, 6.20$
S	$6, 0$	$5.80, 0$	$1, 1$	$.80, 1$
	$6, -.20$	$6.20, .20$	$1, .80$	$1.20, 1.20$

Payoffs under S first-order stochastically dominate payoffs under N.
(Example does not depend on assuming uniform distributions.)

The dominant-strategy prediction is (S,S) with payoffs $(1, 1)$.

	N	S
N	$5, 5$	$0, 6$
S	$6, 0$	$1, 1$

The vertical perspective induces a fog of conflict.

Fog of cooperation

When both players use horizontal projections, they expect:

	W		E	
W	$5,5$	$0,6$	$4.80,5$	$-.20,6$
	$6,0$	$1,1$	$5.80,0$	$.80,1$
E	$5,4.80$	$0,5.80$	$5.20,5.20$	$.20,6.20$
	$6,-.20$	$1,.80$	$6.20,.20$	$1.20,1.20$

Payoffs under E first-order stochastically dominates W.

The dominant-strategy prediction is (E,E) with payoffs $(3.20, 3.20)$.

	W	E
W	$3,3$	$2.80,3$
E	$3,2.80$	$3.20,3.20$

The horizontal perspective induces a fog of cooperation.

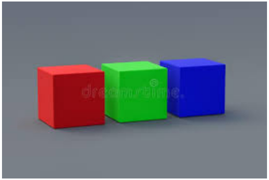
go to end

Language and behavior

Rubinstein (1991) tells a story about how observed regularities in behavior depend on the language employed.

He once set out to test the choice consistency of his baby daughter.

He put Blue, Red, and Green cubes in front of her in an arbitrary order, and she made inconsistent choices.



Until he realized that:

"the baby was actually amazingly consistent. However, she was not consistent in her choice between Blue, Red, and Green. She was consistent in her choice between Left, Center, and Right. She always chose Left."

If his vocabulary had not included the words Left, Center, and Right, he would have been unable to describe this regularity.

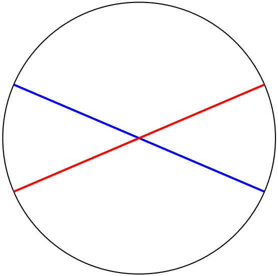
What do you *exactly* mean?

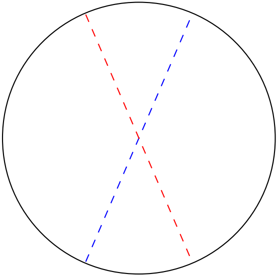
There are two players. Each has a vocabulary with two words, corresponding to a binary partition of the state space.

Primus speaks — and controls — Up and Down.
Secunda speaks – and controls — Left and Right.

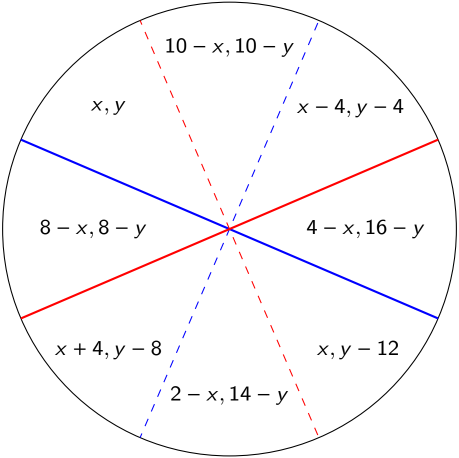
The outcome is random, but conditional on their utterances.

There are two different vocabularies (Blue and Red).
Up and Down are defined by either thick line.
Left and Right are defined by either dashed line.





Each sector is *ex ante* equally likely, with corresponding payoffs.

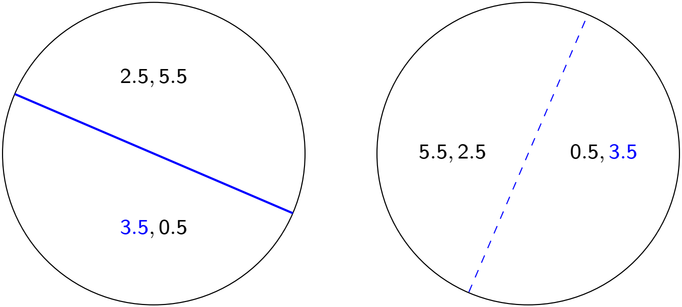


Primus picks one of the thick lines, Secunda one of the dashed lines.

Their choice is based on one of the two vocabularies (Blue or Red).

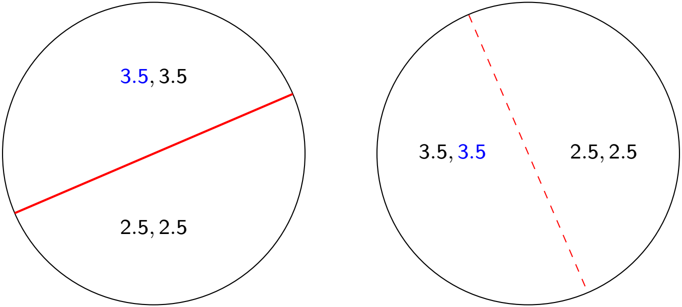
The vocabularies align players' attention.

Under the Blue vocabulary, Primus and Secunda respectively view the circle as



Their preferred options are D^B (Down in Blue) and R^B (Right in Blue).

Under the Red vocabulary, Primus and Secunda respectively view the circle as

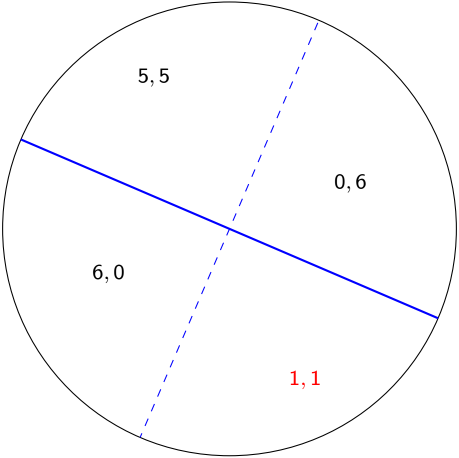


Their preferred options are U^R (Up in Red) and L^R (Left in Red).

The vocabulary affects what players perceive as more advantageous.

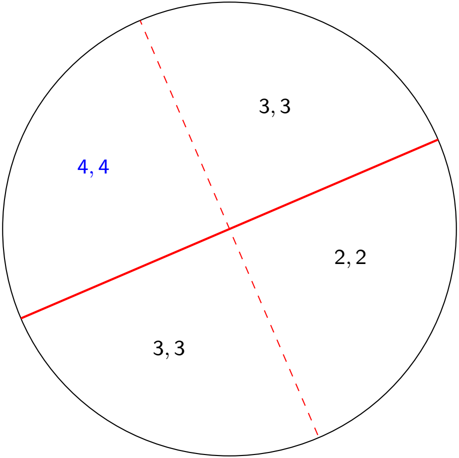
Playing under the same culture

When 1^B and 2^B play, they face a Prisoners' Dilemma.



Blue vocabularies are aligned and elicit a fog of conflict.

When 1^R and 2^R play, they face a trivial Coordination game.



Red vocabularies are aligned and elicit a fog of cooperation.

Either vocabulary induces a different game, that is dominance-solvable.

	L^B	R^B
U^B	$5, 5$	$0, 6$
D^B	$6, 0$	$1, 1$

(Blue)

	L^R	R^R
U^G	$4, 4$	$3, 3$
D^G	$3, 3$	$2, 2$

(Red)

This setup may be framed as a comparison of organizational cultures facing some task assignment: Blue people defect, Red people cooperate.

How does a boss lead Blue players to a Red culture?

> *England and America are two countries separated by the same language.*
>
> attributed to G.B. Shaw

See Churchill's anedocte on *tabling* an issue during negotiations with US.

Conclusions

1. The space of strategies for a player may be richer than he perceives or controls.
2. Subjective models may be self-confirming.
3. Subjective models neglect distinctions, and hence contract players' strategy space.
4. Objective predictions may be amplified in opposite directions: fog of conflict versus fog of cooperation.

... and thank you for your attention!